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## Q-branes in type IIB supergravity

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The notion of Q7-branes and Q-instantons is introduced. These are one-half BPS solutions of the type IIB supergravity theory characterized by a set of three parameters  $p, q, r$  such that  $r^2 > 4pq$ . The numbers  $p, q, r$  parameterize the electric and magnetic couplings of, respectively, the Q7-branes and Q-instantons to a triplet of 8-forms. The  $(p, q)$  7-branes and instantons including the D7-brane and the D-instanton have  $r^2 = 4pq$ . Seven-brane solutions containing Q7-branes are discussed and the Q-instanton on-shell action is computed. The Q7-branes are either novel bound states of a  $(p, q)$  7-brane and a  $(p', q')$  7-brane with  $(p, q) \neq (p', q')$  or they form an independent new object in the theory.

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### 1 Introduction

Recently, 7-brane configurations have been investigated with an emphasis on their supersymmetry properties [1] and their coupling to the bulk IIB supergravity fields [2]. As shown in [1] generic 7-brane configurations contain 7-branes that are associated to various  $SL(2, \mathbb{Z})$  conjugacy classes. Some of these conjugacy classes correspond to  $(p, q)$  7-branes. There appear further 7-branes that are characterized by three parameters  $(p, q, r)$ . The  $(p, q)$  7-branes can be considered to  $(p, q, r)$  7-branes with  $r^2 = 4pq$ . The Q7-branes have parameters  $(p, q, r)$  for which  $r^2 > 4pq$ . For each of these 7-branes one can define an axion, that we denote by  $\chi'$ , with respect to which the 7-brane is magnetically charged.

The objects that are electrically charged under  $\chi'$  and that are dual to the Q7-branes are dubbed Q-instantons [3]. These are half-BPS solutions of Euclidean type IIB supergravity. It is well-known that the object that is dual to the D7-brane is the D-instanton [4]. The Q-instanton source and boundary terms differ from those of the D-instanton. As a result the on-shell actions for the D- and Q-instantons are different. In [5] it was shown that the D-instanton leads to higher order corrections to the string effective action of the form of  $\mathcal{R}^4$  terms. Since Q-instantons preserve the same supersymmetries as the D-instanton, they are expected to contribute to the same  $\mathcal{R}^4$  terms.

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## 2 Type IIB moduli space, axions and 8-forms

Consider the bosonic action for type IIB supergravity on a background with vanishing 3-form and 5-form field strengths described by the well-known axion-dilaton action coupled to gravity

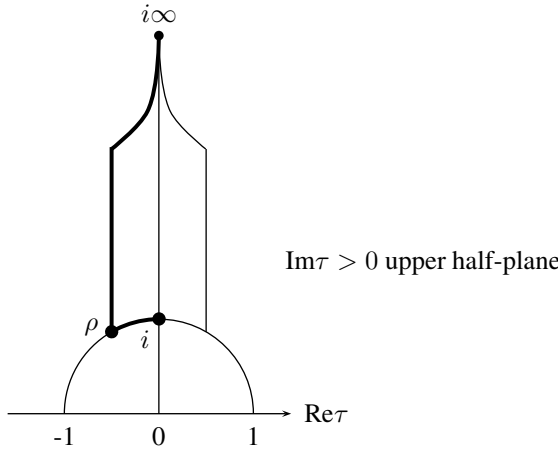
$$S = \int_{\mathcal{M}_{9,1}} \left( \star 1 R - \frac{1}{2} \star d\phi \wedge d\phi - \frac{1}{2} e^{2\phi} \star d\chi \wedge d\chi \right). \quad (1)$$

This action is invariant under non-linear  $PSL(2, \mathbb{R})$  transformations acting on  $\tau = \chi + ie^{-\phi}$  as fractional linear transformations. The classical moduli space of the theory is the set of  $PSL(2, \mathbb{R})$  inequivalent values of  $\tau$ , which is the space  $PSL(2, \mathbb{R})/SO(2)$ .

The quantum moduli space of the inequivalent values of the complex axion-dilaton field  $\tau = \chi + ie^{-\phi}$  of the non-perturbative type IIB string theory is conjectured [6] to be given by the orbifold

$$\frac{PSL(2, \mathbb{R})}{SO(2) \times PSL(2, \mathbb{Z})}. \quad (2)$$

This orbifold is depicted in Fig. 1. The orbifold points are taken to be  $i\infty$ ,  $i$  and  $\rho$ . By  $\rho$  we denote the point  $\rho = -\frac{1}{2} + \frac{i}{2}\sqrt{3}$ .



**Fig. 1** Fundamental domain of  $PSL(2, \mathbb{Z}) \backslash PSL(2, \mathbb{R}) / SO(2)$

Transformations of  $\tau$  under  $PSL(2, \mathbb{R})$  will be written as

$$\tau \rightarrow \Lambda\tau \equiv \frac{a\tau + b}{c\tau + d} \quad \text{with} \quad \Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = e^Q \quad \text{and} \quad Q = \begin{pmatrix} r/2 & p \\ -q & -r/2 \end{pmatrix}. \quad (3)$$

The matrix  $Q$  takes values in the Lie algebra of  $SL(2, \mathbb{R})$ . A point  $\tau_0$  is a fixed point under the  $e^Q$  transformation if it satisfies the equation  $e^Q \tau_0 = \tau_0$ . The fixed points  $\tau_0$  of  $e^Q$  with  $0 \leq \text{Im } \tau_0 < \infty$  and  $q > 0$  for  $\det Q \geq 0$  are given by

$$\tau_0 = -\frac{r}{2q} + \frac{i}{q} \sqrt{\det Q}. \quad (4)$$

In the following it will be assumed that  $e^Q$  takes values in  $SL(2, \mathbb{Z})$ .

The point  $\tau_0 = i\infty$  has  $p = 1$  and  $q = r = 0$  and is the place where the spectrum of D-branes predicted by perturbative type IIB string theory corresponds to brane-like solutions of type IIB supergravity [7]. The  $(p, q)$  7-branes with  $q > 0$  are located on the real axis at  $\tau = -\sqrt{\frac{p}{q}}$ . When we add D-branes (or  $(p, q)$

branes) to the type IIB supergravity theory the duality group  $SL(2, \mathbb{R})$  is broken down to the subgroup that is generated by the shift symmetry of the RR axion (or some  $(p, q)$  transformed axion), *i.e.* the  $\mathbb{R}$  subgroup of  $SL(2, \mathbb{R})$ .

The Q-branes, *i.e.* branes characterized by a triplet of charges  $p, q, r$  such that  $\det Q > 0$ , are associated to the orbifold points  $i$  and  $\rho$  of Fig. 1. In analogy with the D-brane case, when we add a Q-brane to type IIB supergravity the duality group  $SL(2, \mathbb{R})$  is broken down to the subgroup that leaves either  $i$  or  $\rho$  invariant. Such transformations are of the form  $\tau \rightarrow e^Q \tau$  with  $\det Q > 0$  and they form the  $SO(2)$  subgroup of  $SL(2, \mathbb{R})$ .

A convenient basis of the type IIB moduli space  $SL(2, \mathbb{R})/SO(2)$  to describe Q7-branes and Q-instantons is formed by the variables  $T$  and  $\chi'$  defined by

$$\frac{\tau - \tau_0}{\tau - \bar{\tau}_0} = e^{2i\sqrt{\det Q} \mathcal{T}} \quad \text{where} \quad \mathcal{T} = \chi' + \frac{i}{4\sqrt{\det Q}} \log \frac{T + 2\sqrt{\det Q}}{T - 2\sqrt{\det Q}}. \quad (5)$$

As follows from Eq. (5) the values  $\chi'$  and  $\chi' + \pi/\sqrt{\det Q}$  are to be identified and thus  $\chi'$  can be thought of as an angular variable. The requirement that  $\text{Im } \tau > 0$  implies that  $T > 2\sqrt{\det Q}$  or what is the same  $\text{Im } \mathcal{T} > 0$ . For special values of  $p, q$  and  $r$  the point  $\tau_0$  is equal to the points  $i$  and  $\rho$  of Fig. 1.

In terms of the fields  $T$  and  $\chi'$  the action (1) takes the form

$$S = \int_{\mathcal{M}_{9,1}} \left( *1R - \frac{1}{2} \frac{1}{T^2 - 4\det Q} *dT \wedge dT - \frac{1}{2} (T^2 - 4\det Q) *d\chi' \wedge d\chi' \right). \quad (6)$$

This form of the IIB supergravity action is obtained using the field redefinition (5). Next, a 9-form field strength that is dual to  $d\chi'$  is introduced via

$$(T^2 - 4\det Q) d\chi' = * \left( pF_9 + qH_9 + \frac{r}{2} G_9 \right) = *\mathcal{F}_9, \quad (7)$$

where the 9-forms  $F_9, H_9, G_9$  are organized in a triplet transforming in the adjoint of  $SL(2, \mathbb{R})$  and  $p, q$  and  $r$  are the components of the matrix  $Q$ . From the axion  $\chi'$  equation of motion (when ignoring its coupling to the 2-forms and 6-forms) it follows that  $d\mathcal{F}_9 = 0$ , so that locally  $\mathcal{F}_9 = d\mathcal{A}_8$ .

The dynamics of the 8-form  $\mathcal{A}_8 \equiv pC_8 + qB_8 + \frac{r}{2} D_8$  can be described by the following action

$$S[g_{\mu\nu}, \chi', \mathcal{F}_9, T] = \int_{\mathcal{M}_{9,1}} \left( *1R - \frac{1}{2} \frac{1}{T^2 - 4\det Q} *dT \wedge dT - \frac{1}{2} \frac{1}{T^2 - 4\det Q} *\mathcal{F}_9 \wedge \mathcal{F}_9 \right). \quad (8)$$

### 3 Q7-branes

The Q7-brane couples minimally to  $\mathcal{A}_8$  while the Q-instanton couples magnetically with respect to  $\mathcal{A}_8$ <sup>1</sup>. The 7-brane action

$$S = - \int_{\mathcal{M}_8} d^8\sigma T \sqrt{-g_{(8)}} + \int_{\mathcal{M}_8} \mathcal{A}_8, \quad (9)$$

preserves half of the IIB supersymmetries with a projector given by

$$P\epsilon = \frac{1}{2} (1 + i\gamma_{\underline{0}\dots\underline{7}}) \epsilon = \frac{1}{2} (1 - i\gamma_{\underline{8}\underline{9}}) \epsilon = 0, \quad (10)$$

for a 7-brane extended in the directions  $x^1, \dots, x^7$ .

<sup>1</sup> For an early discussion on instantons and monopole-like configurations related to  $(d-2)$ -form gauge fields we refer to [9].

In order to describe 7-brane solutions sourced by Eq. (9) the Killing spinor equations must be solved by imposing the supersymmetry projector (10). The most general 7-brane solution to the type IIB Killing spinor equations is given by [4, 8]

$$ds^2 = -dt^2 + d\vec{x}_7^2 + (\text{Im } \tau)|f|^2 dz d\bar{z}, \quad (11)$$

$$\epsilon = \left(\frac{\bar{f}}{f}\right)^{1/4} \epsilon_0, \quad (12)$$

where the functions  $\tau$  and  $f$  are both holomorphic functions of  $z = x^8 + ix^9$ . The spinor  $\epsilon_0$  is constant and satisfies  $\gamma_{\bar{z}}\epsilon_0 = 0$ . Points  $z \neq \infty$  around which  $(\tau, f)$  has a nontrivial monodromy are the locations of the 7-brane sources (9).

Whenever a branch cut of the function  $\tau$  is crossed it is required that the holonomy of the Killing spinor with respect to the generalized connection appearing in the gravitino supersymmetry variation equation is trivial. This condition requires  $f$  to transform, when crossing a branch cut of  $\tau$ , as [1]

$$f \rightarrow (c\tau + d)f. \quad (13)$$

The function  $f$  thus transforms under  $SL(2, \mathbb{Z})$ . Requiring that the monodromies of  $\tau$  and  $f$  are in agreement with each other fixes the form of the 7-brane solution.

The asymptotic value for  $\tau$  is the coupling constant and when it can be arbitrary it is possible to consider the asymptotic regime as the perturbative starting point for some underlying theory. If for example  $\tau(z = \infty)$  would be equal to  $i\infty$ ,  $i$  or  $\rho$  the solution would have no asymptotic regime in which  $\tau$  can be treated as a free parameter in terms of which a perturbative expansion can be defined. The form of  $\tau$  for arbitrary  $\tau(z = \infty)$  is given by [4, 12]

$$j(\tau) = \frac{P_N}{P_N + Q_N}, \quad (14)$$

where  $P_N$  and  $Q_N$  are arbitrary polynomials of degree  $N$ . The function  $j$  is Klein's modular  $j$ -function that maps the orbifold (2) onto the Riemann sphere. The number  $N$  determines the total number of 7-branes. Writing  $P_N$  and  $Q_N$  as

$$P_N = c_P(z - z_\rho^1) \cdots (z - z_\rho^N) \quad \text{and} \quad Q_N = c_Q(z - z_i^1) \cdots (z - z_i^N), \quad (15)$$

where the points  $z_i$  and  $z_\rho$  are points where  $\tau$  equals  $i$  and  $\rho$ , respectively, and form the locations of the Q7-branes. The parameters  $c_P$  and  $c_Q$  are nonzero complex constants. The general form taken by the function  $f$  is [1]

$$f(\tau) = \eta^2(\tau) (P_N + Q_N)^{-1/12} (z - z_\rho^1)^{-s_1} \cdots (z - z_\rho^N)^{-s_N} (z - z_i^1)^{-t_1} \cdots (z - z_i^N)^{-t_N}, \quad (16)$$

in which  $s_j$  and  $t_j$ ,  $j = 1, \dots, N$ , are as given in Table 1. The function  $\eta(\tau)$  is the Dedekind eta function. Any other monodromy for the pair  $(\tau, f)$  such as  $S^3$  or  $-(T^{-1}S)^4$  can be obtained by combining the monodromies given in Table 1.

The positive values for  $s_j$  and  $t_j$  in (16) lead to negative deficit angles. In order not to have negative energy sources in the solution one must take a sufficient number of Q7-branes coincident with the negative tension objects so that the deficit angle becomes non-negative. It is not necessary to include negative tension objects in order to write down fully well-defined solutions, however, they can be used to undo the presence of certain Q7-branes if desired.

In [2] the Wess–Zumino term for a Q7-brane has been studied and it was found that one needs two Born–Infeld vectors to realize a gauge invariant coupling. This should be contrasted with the case of a  $(p, q)$  7-brane for which only one Born–Infeld vector is required. It is at present not clear what the underlying

**Table 1** Form of the function  $f$  for Q7-branes and negative tension objects.

location	monodromy of $(\tau, f)$	$s_j$ and $t_j$
$z_i^j$	$-S$	$s_j = -1/4$
$\bar{z}_i^j$	$S$	$s_j = 1/4$
$z_\rho^j$	$-T^{-1}S$	$t_j = -1/6$
$\bar{z}_\rho^j$	$T^{-1}S$	$t_j = 1/3$

reason for this is. It might be that the Q7-brane is a bound state of some  $(p, q)$  7-brane with a  $(p', q')$  7-brane with  $(p, q) \neq (p', q')$ , so that it would naturally have two Born–Infeld vectors, but this picture does not (yet) agree well with the fact that there are only two (in static gauge) world-volume scalars describing the motion of the Q7-brane. If one instead focusses on the latter fact and postulates that the Q7-brane is not a bound state but some new independent object in the theory, it is not clear why such an object has never showed up before in studies of type IIB supergravity/string theory.

#### 4 Q-instantons

The Euclidean version of the action (8) is

$$S_E = \int_{\mathcal{M}_{10}} \left( -\star 1R + \frac{1}{2} \frac{1}{T^2 - 4\det Q} \star dT \wedge dT + \frac{1}{2} \frac{1}{T^2 - 4\det Q} \star \mathcal{F}_9 \wedge \mathcal{F}_9 \right). \quad (17)$$

This action can be written as the sum of a quadratic term plus a rest term  $G$  as

$$S_E [T, d\mathcal{A}_8] = \frac{1}{2} \int_{\mathcal{M}_{10}} \frac{1}{T^2 - 4\det Q} \star (dT \mp \star \mathcal{F}_9) \wedge (dT \mp \star \mathcal{F}_9) \pm G, \quad (18)$$

with  $G$  given by

$$G = \int_{\mathcal{M}_{10}} \frac{1}{T^2 - 4\det Q} dT \wedge \mathcal{F}_9 = - \int_{\partial\mathcal{M}_{10}} \frac{1}{4\sqrt{\det Q}} \log \left( \frac{T + 2\sqrt{\det Q}}{T - 2\sqrt{\det Q}} \right) \mathcal{F}_9, \quad (19)$$

where  $\partial\mathcal{M}_{10} = \partial\mathcal{M}_\infty + \partial\mathcal{M}_0$ . The boundaries  $\partial\mathcal{M}_\infty$  and  $\partial\mathcal{M}_0$  are, respectively, the 9-sphere at infinity and around the origin where the field strength  $F_9$  fails to be exact (the location of its magnetic source). By requiring the field  $T$  to blow up at  $|\vec{x}| = 0$  the value of  $G$  is zero at this point and only the boundary at infinity contributes. The first term in the action (18) is positive definite. We thus have the following Bogomol'nyi bound for field configurations respecting the symmetries of the Q(-1)-brane solution

$$S_I \geq \pm G. \quad (20)$$

Solutions that satisfy the Bogomol'nyi bound must have the property that

$$dT = \pm \star \mathcal{F}_9. \quad (21)$$

For such configurations the on-shell value of the action is given by

$$S_E [T, d\mathcal{A}_8] |_{\text{on-shell}} = -G = \frac{\pi|n|}{2\sqrt{\det Q}} \log \left( \frac{T_\infty + 2\sqrt{\det Q}}{T_\infty - 2\sqrt{\det Q}} \right), \quad (22)$$

where  $T_\infty > 2\sqrt{\det Q}$  is the asymptotic value of  $T$ . As shown in [3] the result (22) provides a saddle point approximation of the matrix element of a transition between axion charge eigenstates (or axion conjugate momentum eigenstates).

In order to obtain a saddle point approximation for a matrix element describing the transition between axion field operator eigenstates one must add to (22) the imaginary term [3, 10, 11]

$$-2\pi n i \chi'_\infty, \quad (23)$$

with  $n > 0$  for a Q-instanton and  $n < 0$  for an anti-Q-instanton. The axion that appears in (23) is the axion  $\chi'$  of the Lorentzian IIB theory. The on-shell Q-instanton action thus acquires the form

$$S_E = -2\pi i |n| \mathcal{T}_\infty. \quad (24)$$

In [3] it is shown that the Q-instanton preserves the same supersymmetries as the D-instanton and it is argued that they contribute to the  $\mathcal{R}^4$  corrections that are of order  $\alpha'^3$  with respect to the Einstein–Hilbert term. The coefficient of the  $\mathcal{R}^4$  corrections is an  $SL(2, \mathbb{Z})$  invariant generalized Eisenstein series [5]. When this series is expanded near  $\tau = i, \rho$  it contains an infinite sum of Q-instanton and anti-Q-instanton contributions of the form  $e^{-S_E}$  with  $S_E$  as given in (24). This situation is similar to the case of the D-instanton contributions that emerge when the generalized Eisenstein series is expanded near  $\tau = i\infty$ . There are also non-instantonic terms in the expansions around  $\tau = i, \rho$  whose origin is unclear at present.

## 5 Discussion

The type IIB supergravity theory contains besides the well-known spectrum of  $(p, q)$  branes additionally  $(p, q, r)$  7-branes and instantons, dubbed Q7-branes and Q-instantons when  $r^2 > 4pq$ . The Q7-branes and Q-instantons are naturally associated to the orbifold points  $\tau = i, \rho$ .

The world-volume theory of the Q7-branes is not yet well-understood. One approach to obtaining a better insight might be the following. Consider F-theory on K3 with 24  $(p, q)$  7-branes and force some  $(p, q)$  7-branes with different values for  $(p, q)$  to coincide in such a way that  $\tau$  is not forced to become constant (as happens in the orbifold limits of K3 [13]). Since the monodromy around the composite object can be of the type  $e^Q$  with  $\det Q > 0$  it can be either treated as a Q7-brane or alternatively as a bound state of F-theory 7-branes. Comparing these two interpretations may shed some light on the nature of the Q7-branes.

We believe that the results of [1–3] support the idea that type IIB supergravity provides a valid field theory approximation of some underlying quantum theory near each of the orbifold points of the quantum axion-dilaton moduli space of Fig. 1. It is of interest to understand if in addition to the Q7-branes and the Q-instantons there exist other Qp-brane solutions associated to the orbifold points  $i$  and  $\rho$  of the type IIB quantum moduli space.

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